Some Polynomial Representation Using The (123)-Avoiding Class of The Aunu Permutation Patterns of Cardinality Five Using Binary Codes

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Abstract

Binary codes have interesting applications in digital electronic circuits in which a Boolean variable is used to represent a point in a circuit; hence Boolean algebra can be used as a design tool for digital electronic circuit. Binary codes have been generated using a contagion model reported earlier by the authors. This paper reports a polynomial representation scheme using these binary codes as obtained from the Aunu permutation patterns. The purpose of using polynomial representations is to define operations on the words and sub words generated by Aunu permutation pattern using conventional polynomial arithmetic's except that the coefficients are taken modulo 2 in the completed operations. The polynomials can also be used for construction of mathematical structures such as groups, rings and fields.

KEY WORDS: *Arithmetic operations, Aunu permutation, binary codes, Boolean algebra, Circuits, Mathematical structures, polynomials.*

1. Introduction

Aunu permutation pattern is a class of permutation avoidance which has been found to be of both Combinatorial and Group theoretic importance [1]. The basic similarity between the binary codes and polynomials is that they are an ordered sequence of numbers strung together to represent a single expression. In the case of polynomials, the digits represent the coefficient of each term; in the case of binary codes, the order instead represents the bit's position in the code. Vectors of bits such as message words or code words are represented by binary polynomials in cyclic codes and in finite field of order 2^k i.e GF(2). The coefficients of the polynomials are the bits, and the power of the independent variable is used to denote the position of the bit in the word [2].

1.1 PRINCIPLES OF BINARY ARITHMETIC

Cyclic codes, finite fields of 2^k use binary arithmetic to define operation with signal bits and code word bits. The base operations are addition, subtraction, multiplication and division. Arithmetic is done modulo 2, and the result of addition, subtraction and multiplication simply use ordinary integer arithmetic and take the result modulo 2 [3]. The operations are summarized as follows:

- Binary addition of two bits in the exclusive logical OR of the two bits, usually summarized by mnemonic XOR. The result of an XOR is a zero if the two bits are the same and a one if the two bits are different.

- Binary subtraction is the same as a binary addition because, modulo 2, -1 is the same as +1.
- Binary multiplication is the same as a logical AND of the two bits. The result of an AND is a one if both the bits are ones; the result is zero if either or both of the input bits is zero.
- Division of binary polynomials is based on the fundamental definition of division, which we state here: a number n divided by a denominator d is characterized by a quotient q and a remainder r which are related by $\frac{n}{d} = q + \frac{r}{d}$

Or multiply both sides by the denominator d

$$n = d.q + r$$

Where the remainder r is indivisible by the denominator d. For the purpose of polynomial division, this means that the order of the remainder polynomial r is less than that of the denominator polynomial d [4].

1.2 Algorithm for the derivation of polynomial representation for the (123)-avoiding class of the Aunu permutation patterns of Cardinality five

The following algorithm provides step by step procedures for the derivation of polynomial representation for the (123)-avoiding class of the Aunu permutation patterns of cardinality five:

Step 1: Taking the binary codes from Step 4 of algorithm 3.1.1

Step 2: Building a mapping between binary codes and polynomials i.e

$\sum ax^{j-1}$ *where a is the value of the bit, j is the position* variable.

Step 3: Apply step 2 in to step 1

Step 4: Collect the result of step 3

Step 5: Perform cumulative addition using step 4

Step 6: Perform cumulative multiplication using step 4

Step 7: Perform division on step 4 to find the remainder and quotient of the polynomials generated using the (123)-avoiding class of the Aunu permutation patterns.

1.3 Binary Polynomials Representation of Aunu Permutation Patterns 0f Cardinality Five

$$101 \rightarrow P_1(x) = x^2 + 1 \qquad (1)$$

$$11001 \rightarrow P_2(x) = x^4 + x^3 + 1 \qquad (2)$$

$$1111101 \rightarrow P_3(x) = x^6 + x^5 + x^4 + x^3 + x^2 + 1 \qquad (3)$$

$$1001110001 \rightarrow P_4(x) = x^9 + x^6 + x^5 + x^4 + 1 \qquad (4)$$

$$110000110101 \rightarrow P_5(x) = x^{11} + x^{10} + x^5 + x^4 + x^2 + 1 \qquad (5)$$

Cumulative Addition

$$P_{\sum_{1,2}(x)} = x^4 + x^3 + x^2 \qquad (6)$$

$$P_{\sum_{1,2,3}(x)} = x^6 + x^5 + 1 \qquad (7)$$

$$P_{\sum_{1,2,3,4}(x)} = x^9 + x^4 \qquad (8)$$

$$P_{\sum_{1,2,3,4,5}(x)} = x^{11} + x^{10} + x^9 + x^5 + x^2 + 1 \qquad (9)$$

Cumulative Multiplication

$$P_{\prod_{1,2}(x)} = P_1(x) \times P_2(x) = x^6 + x^5 + x^4 + x^3 + x^2 + 1 \qquad (10)$$

$$P_{\prod_{1,2,3}(x)} = P_{\prod_{1,2}(x)} \times P_3(x) = x^{12} + x^{10} + x^8 + x^6 + x^4 + 1 \qquad (11)$$

$$P_{\prod_{1,2,3,4}(x)} = P_{\prod_{1,2,3}(x)} \times P_4(x)$$

$$= x^{21} + x^{19} + x^{18} + x^{12} + x^{11} + x^{10} + x^5 + 1 \quad (12)$$

$$P_{\prod_{1,2,3,4,5}(x)} = P_{\prod_{1,2,3,4}(x)} \times P_5(x)$$
$$= x^{32} + x^{31} + x^{30} + x^{28} + x^{26} + x^{25} + x^{23} + x^{22} + x^{19} + x^{18} + x^{17} + x^{16} + x^{15} + x^{13} + x^{10} + x^9 + x^7 + x^4 + x^2 + 1 \quad (13)$$

Division Of Polynomials For Aunu Permutation Patterns Of Cardinality Five

Dividing a polynomial with another one of lower degree is similar to normal polynomial division with subtraction as in XOR.

$$101 \to P_1(x) = x^2 + 1$$

$$11001 \to P_2(x) = x^4 + x^3 + 1$$
$$P_{2/1}(x) = 10 \to x \therefore r(x) = x,$$
$$q(x) = x^2 + x + 1 \quad (14)$$

$$1111101 \to P_3(x) = x^6 + x^5 + x^4 + x^3 + x^2 + 1$$
$$P_{3/2}(x) = 0 \quad \therefore r(x) = 0,$$
$$q(x) = x^2 + 1 \quad (15)$$

$$1001110001 \to P_4(x) = x^9 + x^6 + x^5 + x^4 + 1$$
$$P_{4/3}(x) = 10000 \to x^4$$
$$\therefore r(x) = x^4, q(x) = x^3 + x^2 + 1 \quad (16)$$

$$110000110101 \to P_5(x) = x^{11} + x^{10} + x^5 + x^4 + x^2 + 1$$
$$P_{5/4}(x) = 100010011 \to x^8 + x^4 + x + 1$$
$$\therefore r(x) = x^6 + x^4 + 1,$$
$$q(x) = x^2 + x \quad (17)$$

Note: r(x) is the remainder and q(x) is the quotient

Conclusion

The paper reports a polynomial representation scheme using binary codes as obtained from Aunu permutation pattern to define operations on the words and sub words using conventional polynomial arithmetic-except that the coefficients are taken modulo 2 in the completed operations. These binary codes have an interesting applications in digital electronic circuits in which a Boolean variable is used to represent a point in a circuit; hence Boolean algebra can be used as a design tool for digital electronic circuit. Furthermore, the polynomials can be used for the construction of some mathematical structures such as groups, rings and fields.

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